Problem Set 1 - Solution - LV 141.A55 QISS

1. Energy Scales

	Energy	Wavelength	Temperature	Frequency
WLAN	10 μeV	125 mm	115 mK	2.4 GHz
Ambient temperature	26 meV	48 μm	300 K	6.3 THz
Optical Light	2 eV	632 nm	23 kK	470 THz
Ionization energy	24.58 eV	50 nm	280 kK	5.9 10^{15} Hz

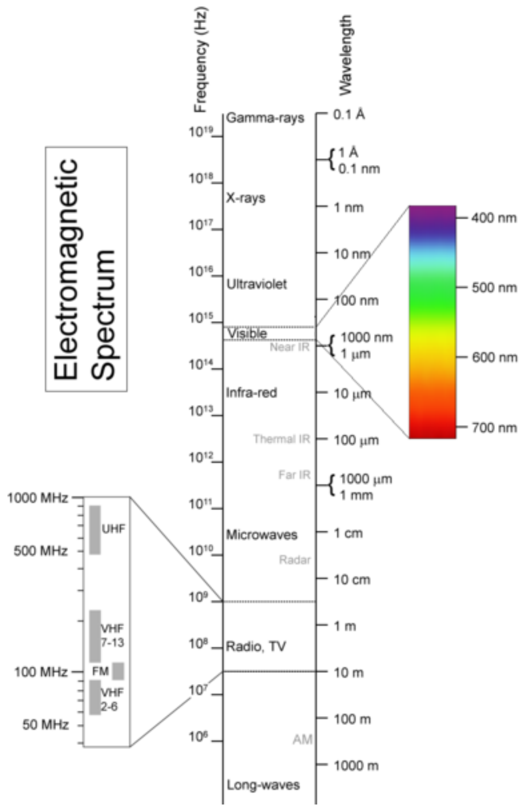

Figure 1: Electromagnetic Spectrum

2. PYTHON - Getting Started

(a)
```
t = linspace(0,10,101)
z = exp(t*(3j-0.5))
plot(t,z.real)
savefig('figure2a.pdf')
```

(b)
```
A=array([[0,1j],[1j,0]])
B=array([[0,1,0],[1,0,1j],[0,-1j,0]])*1/sqrt(2)
C=array([[1,1,1,1],[1,-1,1,-1],[1,1,-1,-1],[1,-1,-1,1]])*0.5

def dag(M):
     return conj(transpose(M))
```

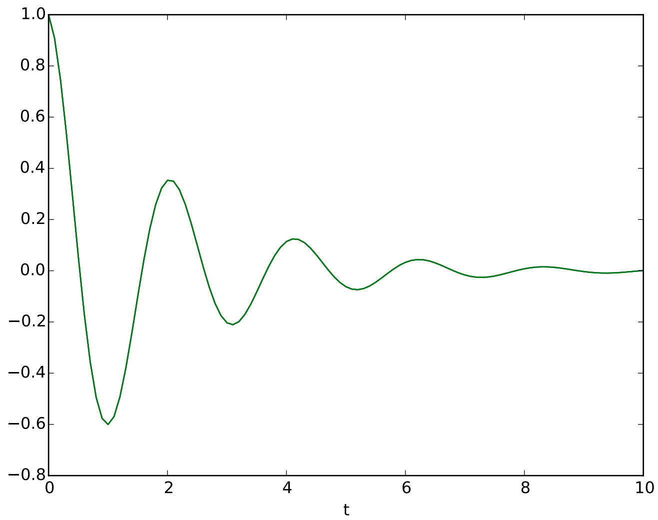

Figure 2: z = exp(t*(3j-0.5))

```
# check if hermitian
A-dag(A))

# check if unitary
dot(A,dag(A))

# trace
trace(A)

# eigenvalues
eig(A)
```

$$A = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

A is not hermitian, but unitary. Trace is 0 and the eigenvalues $i, -i$. A is $i\sigma_x$, the first Pauli matrix times i.

$$B = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

B is hermitian, but not unitary. Trace is 0 and the eigenvalues $-1, 0, 1$

$$C = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

C is unitary and hermitian. Trace is 0 and the eigenvalues $-1, -1, 1, 1$

Note 1: If a matrix is unitary, the eigenvalues have absolute value 1, $|\lambda_i| = 1$. If a matrix is hermitian, the eigenvalues are real.

Note 2: A Hamiltonian H has to be hermitian. A matrix U that transforms one quantum state into another, has to be unitary.